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Nonparaxial Coefficients Influence of High Intense Laser Beam on The Relativistic Self-Focusing Inside Plasma

Fatima Dhari Jabbar[1], Çiğdem Yüksektepe Ataol[1], Ibtisam Jaafer Abd-Ali[2], Munther B. Hassan[2*]
[1]Çankırı Karatekin University, Faculty of Science, Department of Physics, Turkey.
[2]University of Kufa, Faculty of Science, Department of Physics, Najaf, Iraq.
***Corresponding author:** Munther B. Hassan, University of Kufa, Faculty of Science, Department of Physics, Najaf, Iraq, Email: munther.alkhaqani@uokufa.edu.iq

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ABSTRACT

In this article, the influence of the nonparaxial deformation coefficients of laser beam wavefront on the self-focusing phenomenon has been investigated. Due to the relativistic nonlinear interaction between the nonparaxial laser beam and plasma, a self-focused laser beam will have appeared. Appropriate differential equations related to laser beam behavior inside plasma have been derived and then is solved numerically by designing a suitable Matlab program. The result shows that the self-focusing of the laser beam is increasing with decreeing of both the nonparaxial deformation coefficients but the second-order spherical deformation coefficient is greater compared with the spherical deformation coefficient of the fourth order. This study also investigates the influence of the laser beam intensity, plasma frequency, and laser beam diameter on the laser beam self-focusing inside plasma at high enough magnitudes of the laser beam intensity, plasma frequency and laser beam diameter, a high self-focusing of the laser beam will appear. In contrast, the laser beam defocusing phenomenon will be dominant at low magnitudes.

Keywords: *relativistic nonlinearity, laser beam self-focusing, nonparaxial coefficients, plasma density, Gaussian laser beam*

INTRODUCTION

Nowadays more efforts have been focused by researchers on the nonlinear interaction between laser and plasma in both experimental and theoretical scopes [1-4]. Due to the diffraction behavior of laser beams (such as semiconductor injection laser or solid-state laser) through their propagation in media, so more attention has been attracted to investigating the laser beam propagation in the non-paraxial region [5-6]. The self-focusing of laser beam inside media may be considered one of the common and important phenomena.

The designers of laser systems try to avoid the self-focusing effect because it may damage the laser active media [7-9]. In laser plasma nonlinear interactions, the self-focusing effect has been used to arising the primary laser intensity and then to exploit it in many applications such as inertial confinement fusion, generation of high order harmonics, accelerators of charged particles beam by laser plasma coupling, and coherent THz emission [10-12]. Munther and Riyadh studied the self-focusing of the nonparaxial laser beam in magnetized plasma by motivating the ponderomotive nonlinearity.

They found that the increase in the initial laser diameter leading to an increase in the laser beam self-focusing [13].

The article organization will be as following: in section (2) the final formula of the relativistic nonlinear dielectric tensor is calculated. The self-focusing of the nonparaxial laser beam and appropriate equations of deformation coefficients are derived in section (3). The numerical results with a large discussion of the final results and main conclusions are introduced in section (4) and section (5) respectively.

MATERIALS AND METHODS

In this study, Matlab program is used.

The relativistic dielectric constant

The motion equation of an electron inside plasma due to a relativistic laser field is written as

$$m_0 \frac{\partial}{\partial t}\left(\gamma \vec{\upsilon}_0(r,y)\right) = -e\vec{E}_0 \qquad (1)$$

where ($\vec{E}_0$) represents the electric field of the cylindrically symmetric Gaussian laser beam propagating along the z-axis through homogeneous plasma which is given as

$$\vec{E}_0 = \vec{A}_0(r,z)e^{i(\omega_0 t - k_0 z)} \qquad (2)$$

where ω_0 k_0 and $\vec{A}_0(x,z)$ are the angular frequency, wave propagation vector and the amplitude of the electric field respectively.

here $\left(\gamma = (1-\frac{\upsilon_0^2}{c^2})^{-\frac{1}{2}}\right)$ is the relativistic factor which imparts a relativistic oscillation velocity $\vec{\upsilon}_0$ to the plasma electrons which is given as follows

$$\vec{\upsilon}_0 = \frac{ie\vec{E}_0}{m_e \gamma \omega_0} \qquad (3)$$

Depending on equation (2) and equation (3), the relativistic factor γ may be rewritten as [14]

$$\gamma \cong 1 + \frac{1}{2}\cdot\left(\frac{e}{m_0 c\omega_0}\right)^2 A_0 \cdot A_0^* \qquad (4)$$

here m_0, e and c are the electron rest mass, the electron charge and the light velocity respectively.

As a result of the nonlinear interaction of highly intensity laser beam inside the plasma, the effective dielectric constant ε_{eff} of plasma will be modified to become as follows [14].

$$\varepsilon_{eff} = 1 - \left(\frac{\omega_{pe}}{\omega_0}\right)^2 + \frac{1}{2}\left(\frac{e}{m_0 c\omega_0}\right)^2 \left(\frac{\omega_{pe}}{\omega_0}\right)^2 A_0 A_0^* \qquad (5)$$

This equation is consisting of two linear and relativistic nonlinear parts as following

$$\text{Linear part} \quad \varepsilon_0 = 1 - \left(\frac{\omega_{pe}}{\omega_0}\right)^2 \qquad (6)$$

$$\text{Relativistic nonlinear part} \quad \varepsilon_2 A_0 A_0^* = \frac{1}{2}\left(\frac{e}{m_0 c\omega_0}\right)^2 \left(\frac{\omega_{pe}}{\omega_0}\right)^2 A_0 A_0^* \qquad (7)$$

The nonparaxial laser beam self-focusing in relativistic plasma

The general wave equation of the laser electric field through a plasma medium is governed by

$$\nabla^2 \vec{E}_0 - \nabla\left(\vec{\nabla}\cdot\vec{E}_0\right) + \frac{\omega_0^2}{c^2}\underline{\underline{\varepsilon}}\cdot\vec{E}_0 = 0, \qquad (8)$$

Following (Sodha et al.) technique [15] and using Equation (5) so the general wave equation (8) can be rewritten as

$$\frac{\partial^2 A_0}{\partial z^2} + \left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\right)A_0 + \frac{\omega_0^2}{c^2}\left(\varepsilon_0 + \varepsilon_2 A_0 A_0^*\right)A_0 = 0, \qquad (9)$$

Introducing, $A_0 = A_0^0 e^{(ik_0 S)}\, \mathrm{e}^{i(\omega_0 t - k_0 z)}$, where A_0^0 and S are a real function and the phase function of the laser beam inside plasma respectively, therefore Eq.(9) can be separated into real and imaginary parts as follows.

$$2\frac{\partial S}{\partial z} + \left(\frac{\partial S}{\partial r}\right)^2 - \frac{1}{k_0^2 A_0^0}\left(\frac{\partial^2 A_0^0}{\partial r^2} + \frac{1}{r}\frac{\partial A_0^0}{\partial r}\right) = \frac{\varepsilon_2\left(A_0^0\right)^2}{\varepsilon_0}, \qquad (10)$$

$$\frac{\partial\left(A_0^0\right)^2}{\partial z} + \left(A_0^0\right)^2\left(\frac{\partial^2 S}{\partial r^2} + \frac{1}{r}\frac{\partial S}{\partial r}\right) + \frac{\partial S}{\partial r}\frac{\partial\left(A_0^0\right)^2}{\partial r} = 0 \qquad (11)$$

For the sake of more reality, one may adopt the nonparaxial theory as long as the electromagnetic waves undergo a natural diffraction effect through their propagation inside media. In the nonparaxial region, the real function A_0^0 and the phase function S of the laser beam are respectively given as follows [16].

$$A_0^0 = \frac{E_{00}}{f_0}\left(1+\frac{\alpha_{00}r^2}{r_0^2 f_0^2}+\frac{\alpha_{02}r^4}{r_0^4 f_0^4}\right)^{\frac{1}{2}} e^{\left(\frac{-r^2}{2r_0^2 f_0^2}\right)} \quad (12)$$

$$S = \frac{S_{00}}{r_0^2}+\frac{S_{02}r^4}{r_0^4} \quad (13)$$

$$S_{00} = \frac{r^2}{2f_0^2}\frac{\partial f_0}{\partial z} \quad (14)$$

The beam width parameter f_0 represents the variation of laser beam spot size inside plasma through the propagation.

It is useful to mention that the spherical deformation coefficient of the second order α_{00} and the spherical deformation coefficient of the fourth order α_{02} are referring to the spherical curvature of the wavefront and the wavefront departure from the spherical nature respectively. Both α_{00} and α_{02} coefficients are explaining the nonparaxial region contribution in the laser beam self-focusing.

Substituting equations (8, 12, 13, and 14) in equation (10) and equating the coefficients of order r^2 and r^4 of the resulting equation, so one may obtain the following equations

$$\frac{d^2 f_0}{dz^2}=\frac{\left(1-2\alpha_{00}-3\alpha_{00}^2+8\alpha_{02}\right)}{k_0^2 r_0^4 f_0^3}-\left(1-\alpha_{00}\right)\left(\frac{\varepsilon_2 E_{00}^2}{\varepsilon_0}\right)\frac{1}{r_0^2 f_0^2} \quad (15)$$

$$\frac{\partial S_{02}}{\partial z} = \frac{(1-\alpha_{00}+\alpha_{02})\varepsilon_2 E_{00}^2}{2\varepsilon_0 f_0^6} - \frac{(\alpha_{00}^2-7\alpha_{00}\alpha_{02}+\alpha_{00}^2-2\alpha_{02})}{2k_0^2 r_0^2 f_0^6} - \frac{4S_{02}}{f_0}\frac{\partial f_0}{\partial z} \quad (16)$$

last two equations are covering the nonlinear behavior of laser beam in the nonparaxial region through plasma. To solve equations (15 and 16) completely one may use (eq.11) to calculate the variation of the coefficients α_{00} and α_{02} along the z-axis as follows.

$$\frac{\partial \alpha_{00}}{\partial z} = -\frac{16 f_0^2 S_{02}}{r_0^2} \quad (17)$$

$$\frac{\partial \alpha_{02}}{\partial z} = \left(\frac{8 f_0^2 S_{02}}{r_0^2}-\frac{24\alpha_{00} f_0^2 S_{02}}{r_0^2}\right) \quad (18)$$

RESULTS AND DISCUSSION

The discussions of numerical results

This study is based on the nonlinear interaction between the Gaussian mode of Carbon Dioxide (CO_2) pulsed laser, as a pump wave and hydrogen's plasma The final equations (15-18) have been solved numerically using the following set of parameters from experimental observations:

The angular frequency of Carbon Dioxide (CO_2) pulsed laser $(\omega_0 = 1.778\times10^{14}\ rad/\sec)$ corresponding to the wavelength ($\lambda = 10.6\,\mu m$.

The initial laser beam intensities are $((I = 3.26, 5.35, 6)\times10^{20} W/cm^2)$ which are corresponded to the laser strength parameters $(\alpha_r = 0.5, 0.64, 0.68)$.

The initial laser beam diameters are $((x_0 = 30, 44, 48)\,\mu\text{m})$.

The plasma densities are $(n_e = (1, 1.9, 2.3)\times10^{18} cm^{-3})$ corresponding to the plasma frequencies $\left(\omega_p/\omega_0 = 0.32, 0.44, 0.48\right)$ or $(\omega_p = (5.7, 7.8, 8.5)\times10^{13} rad.\sec^{-1})$.

Figures (1) and (2) explain that the increase of the values of the second-order nonparaxial coefficient $(\alpha_{00} = 0.02, 0.04, 0.06)$ and the fourth-order nonparaxial coefficient $(\alpha_{02} = 0.02, 0.04, 0.06)$ are leading to a decrease in the laser beam self-focusing. These results may be interpreted as follows: the wavefront of the laser beam will undergo more curvature as the nonparaxial coefficients are increased thus the focusing ability of the laser beam will become more difficult. Figure (3) illustrates that the laser beam self-focusing in presence of a spherical curvature (represented by α_{00})is greater than the laser beam self-focusing in presence of a nonspherical curvature (represented by α_{02}).

In figures (4, 5 and 6), one may notice the crucial impact of the laser beam intensity, plasma frequency, and laser beam diameter on the laser beam self-focusing respectively. At low magnitudes of these parameters, the diffraction natural term will overcome the focusing term of laser beam (see equation 15). By appropriately increasing these parameter values, namely laser beam intensity, plasma frequency, and laser beam diameter, so one may record a high laser beam self-focusing.

CONCLUSIONS

When a high-intensity laser beam propagates through plasma, it will undergo a relativistic nonlinear interaction. This nonlinear behavior will manifest as a self-focusing phenomenon. The results have explained that the self-focusing of the laser beam in the nonparaxial region is increased strongly with decreasing of deformation coefficients $(\alpha_{00}, \alpha_{02})$. The effect of the spherical deformation coefficient of the second-order (α_{00}) on the laser beam self-focusing is greater compared with the spherical deformation coefficient of the fourth-order (α_{02}). This refers to that the spherical curvature due to (α_{00}) variation is playing a high role in laser beam self-focusing in comparison with nonspherical curvature (α_{02}) variation. The laser beam intensity, plasma frequency, and laser beam diameter have very important effects to control and guide the laser beam propagation inside plasma to give rise to a self-focusing phenomenon.

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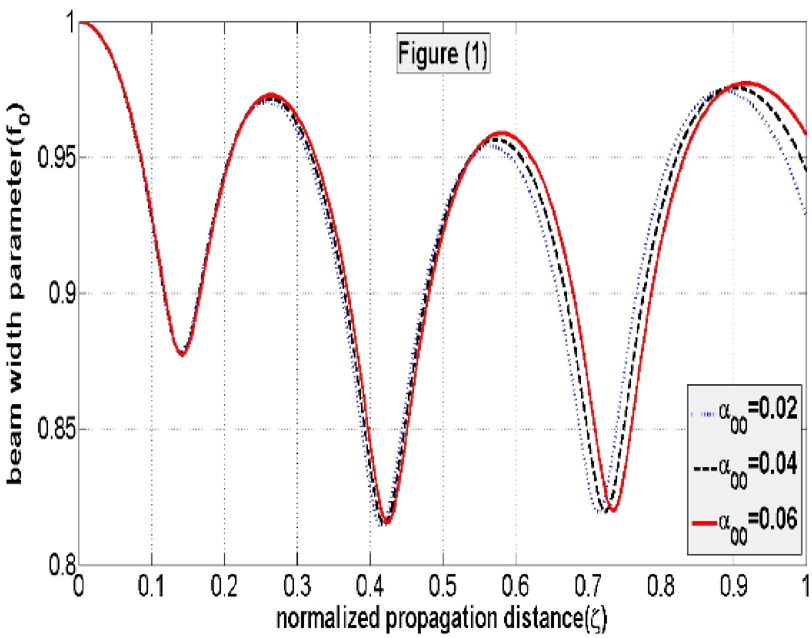

FIG.1: (Color online) Variation of laser beam width parameter (f_0) with normalized distance $\left(\zeta = \frac{z}{k_0 x_0^2}\right)$ at different values of the spherical deformation coefficient of the second-order $(\alpha_{00} = 0.02,\ 0.04, 0.06)$ in nonparaxial region.

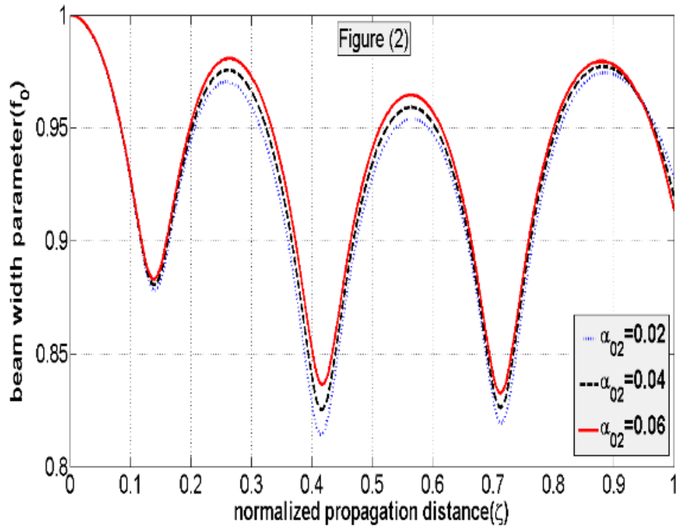

FIG.2: (Color online) Variation of laser beam width parameter (f_0) with normalized distance $\left(\zeta = \frac{z}{k_0 x_0^2}\right)$ at different values of the spherical deformation coefficient of the fourth-order $(\alpha_{02} = 0.02, 0.04, 0.06)$ in nonparaxial region.

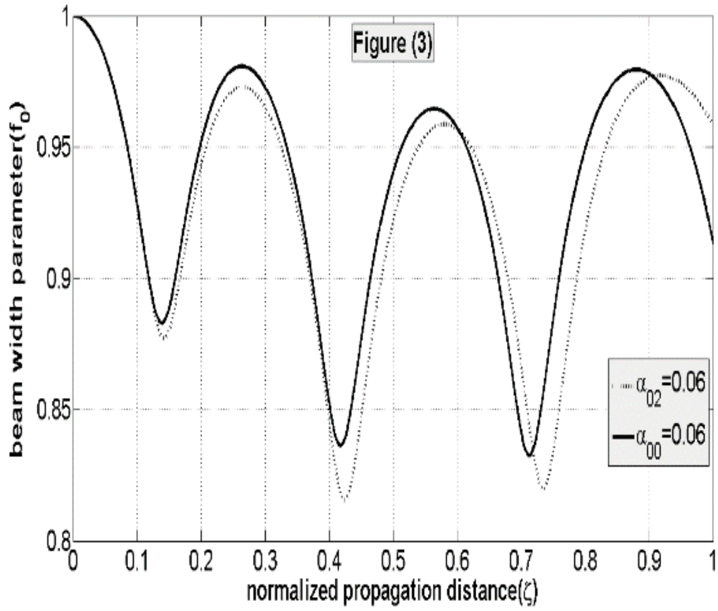

FIG.3: (Color online) Variation of beam width parameter (f_0) with normalized distance $\left(\zeta = \frac{z}{k_0 x_0^2}\right)$. Where sold black line and dotted black line represent $(\alpha_{00} = 0.06)$ and $(\alpha_{02} = 0.06)$ respectively.

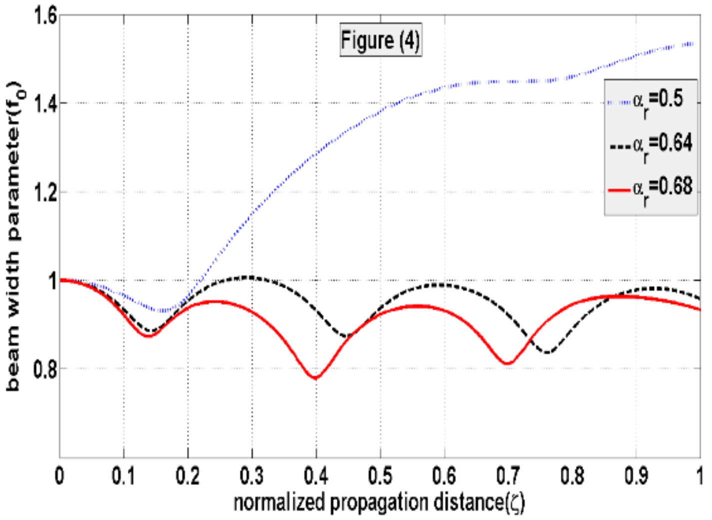

FIG.4: (Color online) Variation of laser beam width parameter (f_0) with normalized distance $\left(\zeta = \frac{z}{k_0 x_0^2}\right)$ at different values of the laser strength parameters $(\alpha_r = 0.5, 0.64, 0.68)$ in nonparaxial region $(\alpha_{00} = \alpha_{02} = 0.02)$.

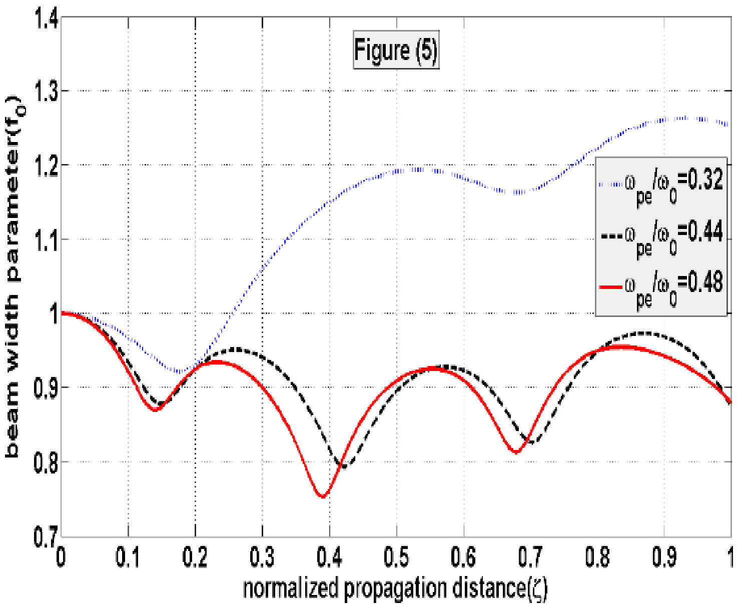

FIG.5: (Color online) Variation of laser beam width parameter (f_0) with normalized distance $\left(\zeta = \frac{z}{k_0 x_0^2}\right)$ at different values of the plasma frequencies $(\omega_{pe} = 0.32\omega_0,\ 0.44\omega_0,\ 0.48\omega_0)$ in nonparaxial region $(\alpha_{00} = \alpha_{02} = 0.02)$.

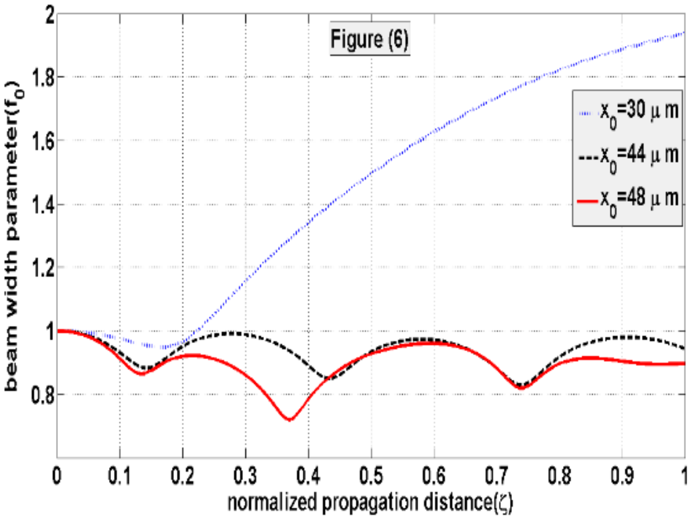

FIG.6: (Color online) Variation of laser beam width parameter (f_0) with normalized distance $\left(\zeta = \frac{z}{k_0 x_0^2}\right)$ at different values of the laser beam diameters $(x_0 = 30\mu\text{m}, 44\mu\text{m}, 48\mu\text{m})$ in the nonparaxial region $(\alpha_{00} = \alpha_{02} = 0.02)$.